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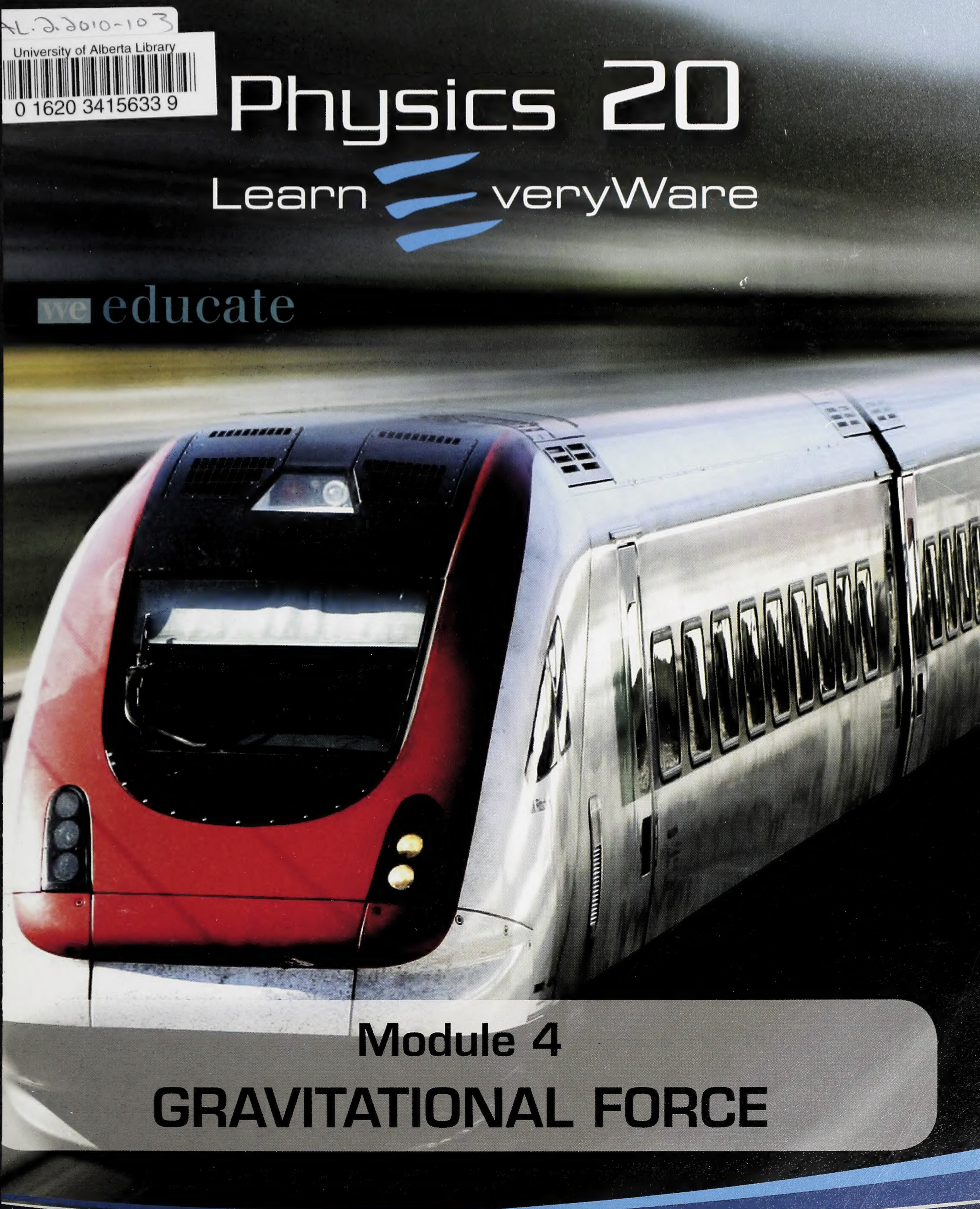

Module 4
GRAVITATIONAL FORCE

Rocky View School Division
Red Deer Catholic Regional Schools
Edmonton Public Schools
Pembina Hills Regional School Division No. 7
Black Gold Regional Schools
Peace Wapiti School Division No. 76
Calgary Board of Education
Alberta Education

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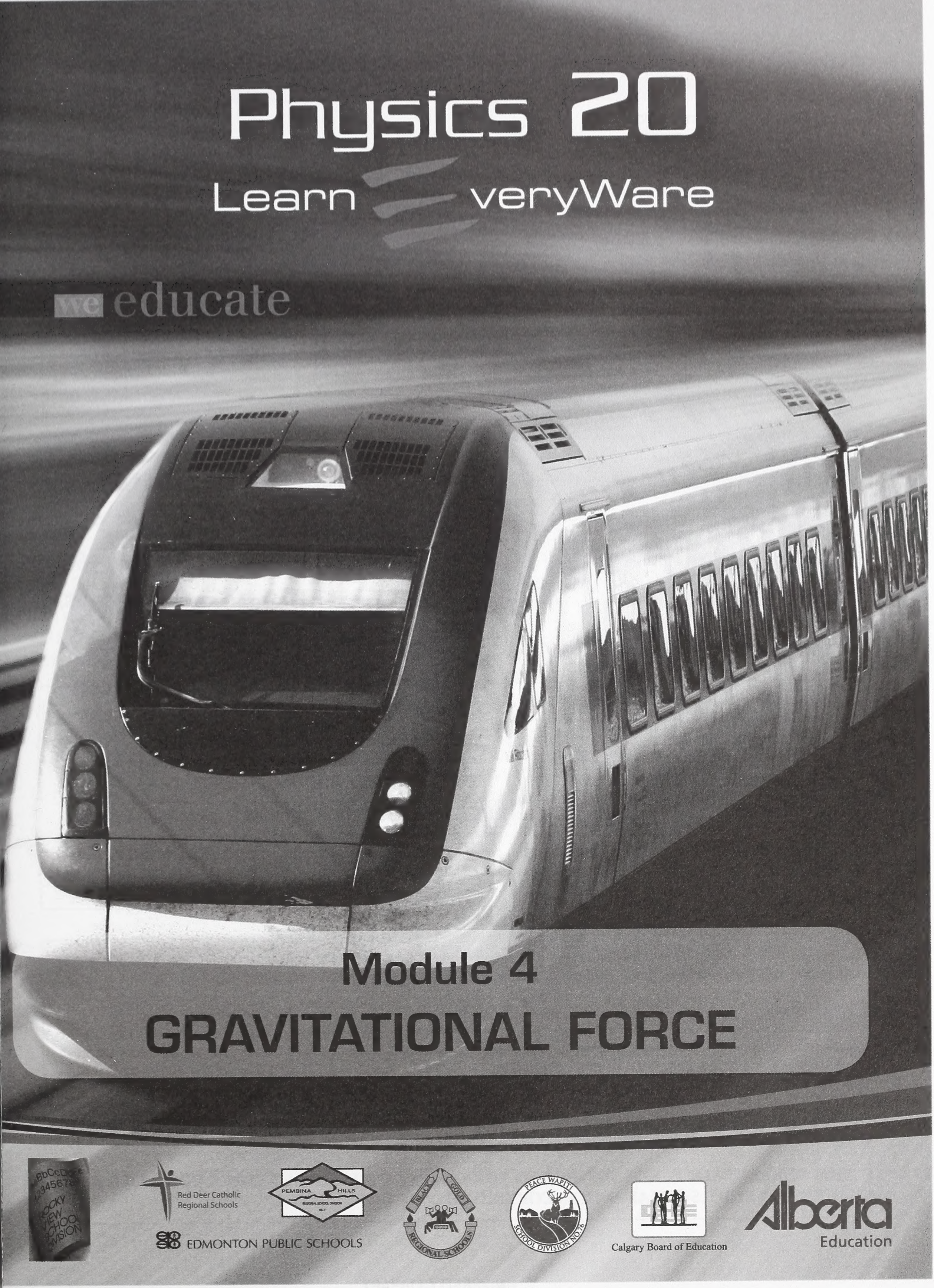

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- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton School District No. 7
- Peace Wapiti School Division No. 76
- Pembina Hills Regional Division No. 7
- Red Deer Catholic Regional Division
- Rocky View School Division No. 41

Physics 20
Module 4: Gravitational Force
Student Module Booklet
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Teachers	✓
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Home Instructors	
General Public	
Other	

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- Alberta Education, **http://www.education.gov.ab.ca**
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- Tools4Teachers, **http://www.tools4teachers.ca**

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Module 4—Gravitational Force

Module Introduction

In Module 4 you will study gravitational forces and gravitational fields. You will learn about Newton's law of universal gravitation and the impact that it has on the world and how people interact with it.

The essential question that you will be looking at in this module is this:

- What role do gravitational effects play in the universe?

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Big Picture

Image courtesy of the Image Science & Analysis Laboratory, NASA Johnson Space Center

Have you ever considered using the Moon, planets, and stars as part of your playground? In many ways, Homo sapiens have been imagining this for centuries. The reality is much more recent. The picture to the right shows Earth rising over the Moon's horizon. It was taken from the *Apollo 11* spacecraft. Humanity stretched its playground by about 385 000 km, and men walked on the Moon's surface on July 20, 1969. Does anyone in your family remember what they were doing that day? Check with your family and friends' families to see how humanity's view of the world and space were changed by that event. The last manned trip to the Moon took place three years later. Since then, the exploration of the solar system and the universe has taken place either closer to home or by using remotely controlled spacecraft.

Image Courtesy of NASA, ESA, and The Hubble Heritage Team (STScI/AURA)

One of these spacecraft, the *Hubble Space Telescope*, took the image to the left in 2004. This spectacular image is of Barred Spiral Galaxy NGC 1300. NGC 1300 is 69 million light-years away and is 110 000 light-years wide. The universe is one mighty big playground.

Can you imagine the knowledge needed to successfully launch a spacecraft and land it on the Moon or the knowledge needed to explain the shape and movement of NGC 1300?

You are about to look at the foundations of that knowledge, and again it begins with Isaac Newton and his discoveries in mathematics and science. In Module 4 you will learn about Newton's law of universal gravitation and the ways in which it explains the universe. In this module you will be exploring the following question: What role do gravitational effects play in the universe?

As you work through Module 4, keep the following essential questions in mind:
- How does gravity vary relative to distance between objects?
- How are weight and mass different?
- How are weight and the force of gravity related?
- How does gravity affect planets, moons, and other objects in the solar system?
- What does the law of universal gravitation tell you about gravity?
- Is the force of gravity a constant?
- How do scientists calculate weight on other planetary objects, such as the Moon?
- How does the law of universal gravitation explain the motion of orbiting bodies?
- How can you calculate the mass of Earth?

These questions should help you fit your world into our amazing universe and the physics that helps explain it all.

In addition to your lesson work you will have a module assessment to complete. For more information about the module assessment and evaluation criteria, go to the Module Assessment section of In This Module.

In This Module

Lesson 1—Acceleration Due to Gravity

In this lesson you will study how Earth's gravitational field affects falling bodies near Earth's surface. You will also study how the strength of this field changes with distance from Earth's surface.

As you work through this lesson, you will explore the following questions:
- How does gravity vary relative to distance between objects?
- How are weight and mass different?
- How are weight and the force of gravity related?
- How does gravity affect planets, moons, and other objects in the solar system?

Lesson 2—Newton's Laws of Universal Gravitation

In this lesson you will learn more about Earth's gravitational field. You will learn that gravity is a universal force between objects, and you will learn how to calculate this force.

You will explore the following essential questions:
- How does gravity affect planets, moons, and other objects in the solar system?
- What does the law of universal gravitation tell you about gravity?
- Is the force of gravity a constant?

Lesson 3—Universal Gravitational Field Theory and the Density of Earth

In this lesson you will learn how the universal gravitational constant can be calculated. You will also study how to "weigh Earth." You will see how these measurements can help you better understand your world.

You will explore the following essential question:
- How did Kepler's laws help calculate the position of the planets as they orbited the Sun?

Module 4 Assessment

The assessment for Module 4 consists of three (3) assignments, as well as a final module inquiry project.
- Module 4: Lesson 1 Assignment
- Module 4: Lesson 2 Assignment
- Module 4: Lesson 3 Assignment
- Module 4 Project

Module 4 Project

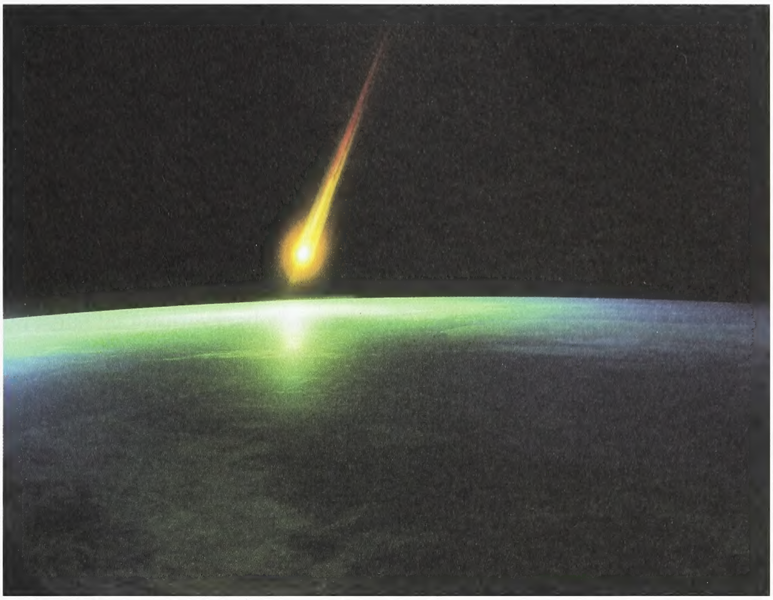
The Module 4 Project is to choose **one** of the following items from your Physics 20 course folder:
- Going Beyond item from Lesson 1
- Reflect on the Big Picture item from Lesson 2
- Reflect on the Big Picture item from Lesson 3

You are then to submit your one item to your teacher for grading. Along with the item from the course folder, include an explanation about why you chose that particular item to submit.

Lesson 1—Acceleration Due to Gravity

Get Focused

Every year, an estimated 500 meteorites reach the surface of Earth. At impact, they generally range in size from a marble to a basketball, but they can be much larger. Nearly all of the meteorites that land on Earth are called chondrites, which are thought to have originated in the solar system's asteroid belt (a vast collection of material that did not form larger objects such as planets or moons). They are believed to be about 4.55 billion years old! Other meteorites resemble the lunar rocks retrieved during the *Apollo* missions to the Moon, suggesting some material reaching Earth could originate from other planets or moons.



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Why are these objects falling to Earth? Why can you look up into the night sky and see shooting stars streaking across the sky and a satellite cruising by, but only one of them is falling? Is there a contradiction here? The answers to these questions will be found by examining the nature of Earth's gravitational field.

In this lesson and related simulation you will explore the following essential questions:

- How does gravity vary relative to distance between objects?
- How are weight and mass different?
- How are weight and the force of gravity related?
- How does gravity affect planets, moons, and other objects in the solar system?

Module 4: Lesson 1 Assignments

In this lesson you will complete the Lesson 1 Assignment in the Module 4 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, and TR 4

The other questions in this lesson are not marked by the teacher; however, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all the questions in this lesson and place those answers in your course folder.

Explore

gravity: a force of attraction towards the centre of a mass

Gravity was studied by Galileo and Galileo, and exploited by many. Common experiences with the force of gravity include some exciting activities and also a few painful ones. You probably started to learn about gravitational effects at an early age from the playground slide, swing, and teeter totter. Later in life, you may have learned more painful lessons, such as falling out of trees or crashing down ski slopes. Less painful activities could have included rock climbing and riding in elevators. From these experiences, you know that gravity is a force that causes objects to fall to the surface of Earth. With more detailed studies, scientists found that objects near the surface of Earth tend to fall at exactly the same rate, with constant acceleration toward the centre of Earth. This is often described as the law of falling bodies, and it seems to contradict the personal observation that snowflakes appear to fall at a much slower rate than rain drops.

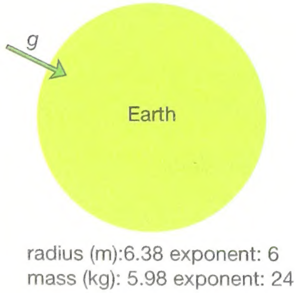
Watch and Listen

In the absence of all other variables, such as air, the law of falling bodies is easily demonstrated both on Earth and on the Moon. The Law of Falling Bodies simulation demonstrates this law in a variety of conditions. Go to your Physics 20 Multimedia DVD, and watch the "Law of Falling Bodies" video.

Earth's Gravitational Field

gravitational field: a three-dimensional region of attraction of other masses created by any object with mass and extending outward in all directions

The **gravitational field** (three-dimensional region of influence) extending outward from Earth is a vector field. (Vector fields associate a length, or *magnitude*, as well as a *direction* to every point in space.) For your purposes here, just remember that the gravitational force has both magnitude and direction everywhere in the vicinity of Earth.



A simulation will be used to visualize the characteristics of a gravitational field, such as the one produced by Earth.

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "inverse square law" in the search bar. Choose the second item (applet) called "Inverse Square Law (Grades 11 and 12)."

mass: the physical amount of matter that makes up an object

The default settings on the simulation show the radius (6.38×10^6 m) and **mass** (5.98×10^{24} kg) of Earth, as well as the gravitational field direction and strength.

Module 4: Lesson 1 Assignment

Remember to submit the answers to TR 1, TR 2, and TR 3 to your teacher as part of your Lesson 1 Assignment in your Module 4 Assignment Booklet.

Try This

TR 1. On the simulator, click the "Vector" button (✓), and then click anywhere in the display to see the gravitational field vector (*g*). Click and hold to drag it around Earth. You will notice two general characteristics of the gravitational field.

a. As you move closer to Earth, the gravitational field becomes _____________, as illustrated by the length of the vector.

b. The gravitational field is always directed ____________ the centre of Earth.

c. The largest value for *g* (acceleration due to gravity), as displayed in the data table, is __________ and occurs at the ___________ of Earth. (**Note:** The units for *g* (N/kg), as displayed in the applet, are identical to the more familiar unit for acceleration (m/s^2). You will learn more about this later in the lesson.)

Note that the "E" in the data table refers to the "Exponent for a power of 10." For example, 8.15E6 m is written in conventional notation as 8.15×10^6 m.

You should notice that the field strength (acceleration due to gravity) increases as you approach the mass (Earth) and that it is at a maximum when you reach Earth's surface. Exactly how much it increases depends on the distance from the mass. You will use graphical analysis to determine the exact mathematical relationship between the field strength (acceleration due to gravity) and the distance from the mass causing it.

TR 2. Using the simulator, experimentally determine how the gravitational field strength (*g*) varies with the distance from Earth (*d*).

a. Click on a point near Earth's surface. For this point, record the following values in the table using conventional notation:

- the distance (*d*), measured from the centre of Earth
- the gravitational field strength (*g*) at this point

Click and drag the field vector farther away from the surface. Record the new distance and gravitational field strength (*g*) at this point. Complete the data table below.

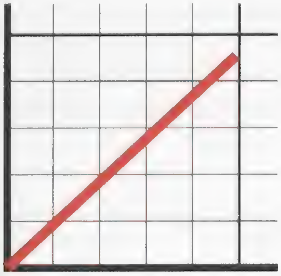
Distance (*d*) m	**Gravitational Field Strength (*g*) N/kg**

b. Choose an appropriate scale and plot the data on the graph provided. The manipulated variable (*d*) is on the *x*-axis, and the responding variable (*g*) is on the *y*-axis.

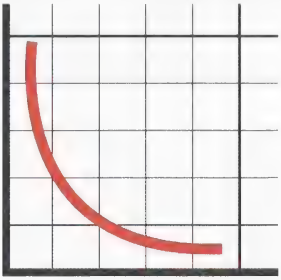
Gravitational Field Strength vs. Distance

TR 3. Which mathematical relationship best describes your graph of gravitational field strength versus distance? State the relationship using *g* for gravitational field strength and *d* for distance from source.

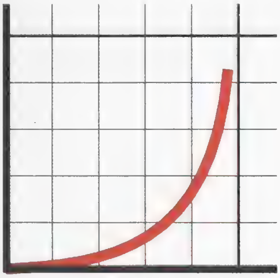
A. relationship: $y \alpha x$



B. relationship: $y \alpha \frac{1}{x^2}$



C. relationship: $y \alpha x^2$



You have just stated the **inverse square law** that describes a gravitational field.

inverse square law: any physical law stating that some quantity is inversely proportional to the square of the distance from a point

Read

Read "Describing Gravitational Force as a Field" on pages 200 and 201 in your physics textbook to reinforce what you learned from your work in the previous simulation.

Try This

You hear the terms *mass* and *weight* incorrectly used as synonyms for each other. Enhance your understanding of the differences between them in the following activity.

Module 4: Lesson 1 Assignment

Remember to submit the answer to TR 4 to your teacher as part of your Lesson 1 Assignment in your Module 4 Assignment Booklet.

TR 4. Complete “4-2 QuickLab: Relating Mass and Weight” on page 198 in your textbook. Obtain the needed materials from your teacher. Be sure to zero the spring scale in the upright position before you start to take measurements. Let mass (in kg) be the manipulated variable, and plot it on the horizontal axis of your graph.

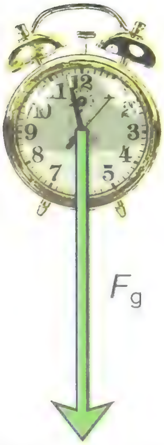
The Concept of Weight and the Force Due to Gravity

The terms *weight*, *mass*, and *acceleration due to gravity* are all related but are often misused. A common example of this is a weigh scale, which is really a device designed to report an object’s mass in kilograms, not its weight in newtons. Without weight, however, a scale won’t work. So what is the relationship? An application of Newton’s second law and some free-body analysis will reveal the connections between mass and weight. You can review free-body diagrams in Lessons 4 and 5 of Module 3.

Suppose you knock your alarm clock off the night stand when the alarm starts buzzing. The clock would fall under the influence of gravity. Without considering air resistance, the following free-body diagram describes the forces acting on the clock.

According to Newton’s law, the net force acting on the clock is

$$\vec{F}_{\text{net}} = \vec{F}_{\text{g}}$$
$$m\vec{a} = \vec{F}_{\text{g}}$$



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Since the clock is accelerating due to gravity, then $a = g$, where $g = -9.81\ \text{m/s}^2$ at Earth's surface. Thus, the equation becomes

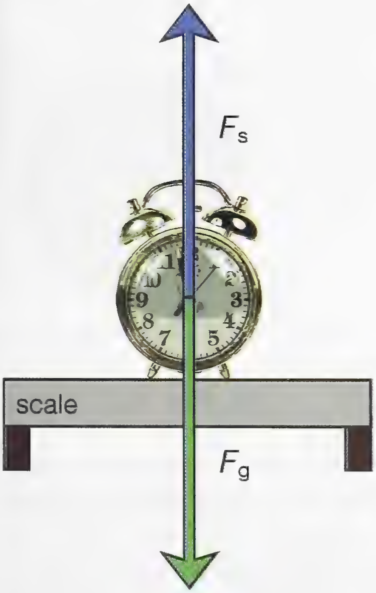
$$\vec{F}_g = m\vec{g}$$

- The gravitational force that acts on the clock is $\vec{F}_g$ and represents its weight. $\vec{F}_g$ may also be written as $\vec{W}$ to represent weight ($\vec{W} = m\vec{g}$).
- The mass of the clock is represented by m, and $\vec{g}$ is the acceleration due to gravity (always pointing down).

If the clock were placed on a scale, the springs in the scale would produce an upward force on the clock equal in magnitude to its weight $\left(\left|\vec{F}_g\right|\right)$. Therefore, the spring scale actually measures the object's weight, not its mass. However, the mass may be displayed by simply dividing out the acceleration due to gravity, which can be done electronically on a digital scale or by adjusting the numbers on a rotating display dial.

$$F_g = mg$$

$$m = \frac{F_g}{g}$$



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It should be noted that the mass of an object does not change even if its weight changes as a result of moving it away from the surface of Earth (or any other mass in the universe). In addition, the units for the acceleration due to gravity can be reported in one of two ways.

Method 1

$$g = \frac{F_g}{m}$$

$$g = \frac{\text{N}}{\text{kg}}$$

Method 2

since $1\ \text{N} = 1\ \text{kg}\bullet\text{m/s}^2$

$$g = \frac{\text{N}}{\text{kg}}$$

$$g = \frac{\text{kg}\bullet\text{m/s}^2}{\text{kg}}$$

$$g = \text{m/s}^2$$

Read

Read "The Concept of Weight" on pages 197 to 199 in your physics textbook.

Self-Check

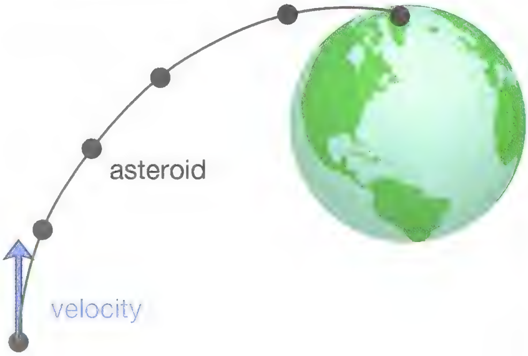
SC 1. Complete the "Concept Check" question on page 197 in your physics textbook.

SC 2. Complete the "Concept Check" question on page 201 in your physics textbook.

Check your work with the answer in the appendix.

Reflect and Connect

Is it possible for an asteroid to pass near Earth and be "captured" by the gravitational field, causing it to follow the path illustrated here? Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "weight and orbits" in the search bar. Choose the item called "Weight and Orbits." Use the simulator to test this scenario. Consult the Show Me, if needed.



- After the simulator window opens, click the "Reset" button on the lower right of the window. Then set the scale to 100 px = 10 000 km ( ).
- Reposition the asteroid—it is at the rear of the velocity vector arrow—by dragging it to where you want it.
- Adjust the velocity vector by dragging the tip to a new position or changing the numbers in the boxes and pressing "Enter." Experiment with different velocities and distances, and view the weight, acceleration, and velocity vectors.
- Sketch the acceleration, velocity, and weight vectors on each of the yellow dots along the asteroid's path in the image.

How does the gravitational field of Earth affect the magnitude and direction of the asteroid's velocity? Place your answer to Reflect and Connect in your Physics 20 course folder.

Discuss

Using the information from the Reflect and Connect section, consider the mass and weight of the asteroid at each of the yellow points on the path. (View the weight vector on the simulator.)

D 1. Which quantity changes as it approaches Earth?

D 2. Why does this happen?

Post your answers to D 1 and D 2 to the discussion area. Consider at least two other responses and edit your own responses, if needed. Submit your final reflection to your instructor for feedback.

Going Beyond

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NASA's Near Earth Object (NEO) program defines an *object* as a comet or asteroid that has been nudged by the gravitational attraction of nearby planets, putting it into an orbit that may meet Earth at some point in the future.

As of 2007, NASA had identified 4559 NEOs that pose varying degrees of threat to the inhabitants of Earth, should these objects eventually crash into the planet. For example, the 400-m wide asteroid called 2004 MN4 is predicted to pass near Earth on April 13, 2029.

The odds of impact, based on observations up to 2007, are 1 in 300, which is deserving of special observation by astronomers. MN4 is the first object to receive a rating of 2 on the Torino scale. As NASA expands its searches, more objects will likely be found making a close but not unusual pass near Earth.

Research the purpose of the Near Earth Object program. Prepare a short report that discusses your findings. You should include responses to questions such as the following in your report:

- Is it possible to identify every threat?
- What is the Torino scale?
- How much time, money, and energy should be devoted to such a program?
- If we were informed of an upcoming, catastrophic collision, what would we do?
- Should the countries of the world be developing an impact prevention plan?
- What do you see this program consisting of?
- What are the cost and the benefits to the program?

Add this report to your Physics 20 course folder.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider travel to, exploration of, or study of space. To help you reflect on your learning from this lesson, complete at least one of these reflection activities.

- When *Sputnik* was launched on October 4, 1957, the world changed and the space race began. Explain.
- Create an illustration of the launch and orbit of *Sputnik*.
- Research space junk and satellites falling out of orbit.
- Create a multimedia presentation that includes photos (either your own or from the Internet) to illustrate how acceleration affects the orbits of satellites.

Store your completed reflection in your Physics 20 course folder.

Module 4: Lesson 1 Assignment

Make sure you have completed all of the questions for the Lesson 1 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 4 assignments have been completed.

Your teacher may also require you to submit your Discuss questions and the report you wrote in the Going Beyond section either for marks or feedback.

Lesson Summary

In this lesson you explored the following questions:

- How does gravity vary relative to distance between objects?
- How are weight and mass different?
- How are weight and the force of gravity related?
- How does gravity affect planets, moons, and other objects in the solar system?

The law of falling bodies states that objects near Earth's surface will fall at exactly the same rate with constant acceleration. This acceleration (g) is caused by Earth's gravitational field—a three-dimensional region of influence surrounding the planet. The field strength (g) increases as you approach the mass—Earth, in this context—and it is at a maximum at the surface ($-9.81\ \text{m/s}^2$). The field strength (g) obeys the inverse square law for a gravitational field ($\left(g \propto \frac{1}{d^2}\right)$) and extends outwards, affecting the motion of objects that encounter it. These objects include planets, moons, and artificial satellites.

The terms *weight* and *mass* are often confused and used incorrectly. Mass is a measure of the amount of matter that makes up an object, whereas an object's weight is the force of gravity on the object. The magnitude of this force is equal to the product of the object's mass and the acceleration due to gravity at its location. Weight is a measure of the force of gravity on an object. Note that weight is based on Newton's second law ($\vec{F} = m\vec{a}$), where the acceleration is due to gravity. Expressed as an equation, it is as follows:

$$\vec{W} = m\vec{g}$$

Quantity	Symbol	SI Unit
weight	$\vec{W}$	N
mass	m	kg
acceleration due to gravity	$\vec{g}$	m/s^2

* The acceleration due to gravity at Earth's surface averages around $-9.81\ m/s^2$.

Lesson Glossary

gravitational field: a three-dimensional field created by any object with mass and extending outward in all directions, influencing other masses

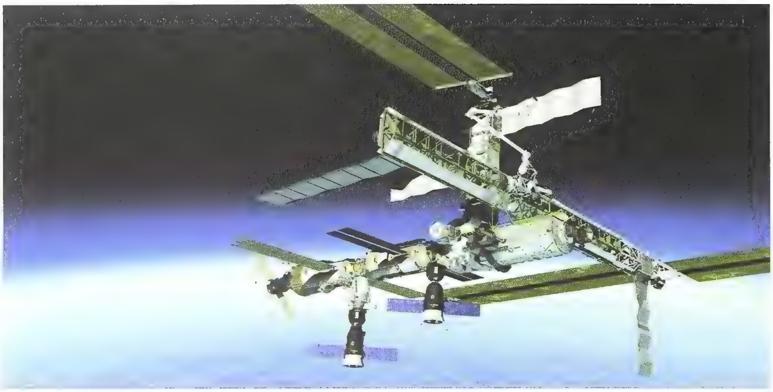
gravity: a force of attraction towards the centre of a mass

inverse square law: any physical law stating that some quantity is inversely proportional to the square of the distance from a point

mass: the physical amount of matter that makes up an object

Lesson 2—Newton's Law of Universal Gravitation

Get Focused



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The International Space Station (ISS) is a research facility and joint project of five space agencies including the Canadian Space Agency. The station continues to be constructed in orbit and has been continually occupied by humans since the first crew arrived on November 2, 2000. It is also an artificial satellite, held in orbit by Earth's gravitational field.

This is the same field that keeps smaller pieces of "space junk" in orbit—in addition to much, much larger objects such as the Moon. Considering this, what do the Moon, space junk, and the ISS have in common? Why are they all attracted to Earth in a similar, universal way?

Evidence of gravitational fields is common throughout the universe. For example, objects on the surface of Earth fall to the ground if they are not supported. Satellites and moons orbit their respective planets. In our solar system, all the planets orbit the Sun. In deep space it is possible to see stars pulling on and orbiting one another within a region of influence. Mass is common in all of these examples. For this reason, a gravitational force is often referred to as a mass-attracting force. It is one of the fundamental forces in nature and exists between any two masses. Understanding this force can help you understand why objects accelerate at the rate they do. For example, a falling apple, the International Space Station, and the Moon are all influenced by Earth's gravitational field. Therefore, they are all accelerating. But do they accelerate at the same rate? Is the Moon falling just like the apple? Is the space station falling too?

In this lesson and related lab activities you will explore the following questions:

- How does gravity affect planets, moons, and other objects in the solar system?
- What does the law of universal gravitation tell you about gravity?
- Is the force of gravity a constant?

Module 4: Lesson 2 Assignments

In this lesson you will complete the Lesson 2 Assignment in the Module 4 Assignment Booklet.

- Lab—LAB 1, LAB 2, and LAB 3
- Try This—TR 1, TR 2, TR 3, and TR 4
- Reflect and Connect

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the "Law of Universal Gravitation" video.

Explore

The space station isn't falling—at least, it isn't falling in the traditional way you think of things falling such as an apple from a tree. Yet the space station and the apple are both accelerating in the same gravitational field produced by the same mass, Earth. Recall Newton's second law, which will help you understand the universal nature of the force of gravity.

Newton's Second Law

The sum of all forces, $\vec{F}_1, \vec{F}_2, \vec{F}_3, \cdots$, acting on a particle is equal to the product of the particle's mass (*m)* and the particle's acceleration ($\vec{a}$), as illustrated in the following equation.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$$\vec{F}_{net} = m\vec{a}$$

$$m\vec{a} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

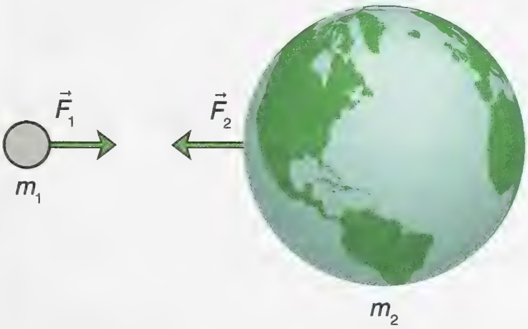
How does this equation apply to **Newton's law of universal gravitation?**

Newton's law of universal gravitation: a law that states the gravitational force of attraction between any two masses is proportional to the product of the two masses and inversely proportional to the square of the distance between them

It is summarized mathematically as

$$F_g = \frac{Gm_1 m_2}{r^2}$$

Newton's Law of Universal Gravitation



Any two particles exert attractive gravitational forces on each other. The illustration shows two masses, m_1 and m_2, each consisting of billions of particles, and their attractive gravitational forces, $\vec{F}_1$ and $\vec{F}_2$. Each of these forces is the vector sum of the attractive forces of all the particles in the huge mass on the other mass. *Mass*$_1$ exerts a force on *mass*$_2$ that is directed towards *mass*$_1$ and vice versa. Both forces have the same magnitude $\left(\left|\vec{F}_1\right| = \left|\vec{F}_2\right|\right)$.

The equation $F_g = \frac{Gm_1m_2}{r^2}$ describes the magnitude of the attractive gravitational force, where r is the separation between the two masses and G is the **universal gravitational constant**. The value of the universal gravitational constant is

$$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg}\bullet\text{s}^2}$$

or

$$G = 6.67 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$$

universal gravitational constant: the value G in the equation $F_g = \frac{Gm_1m_2}{r^2}$ summarizing Newton's law of universal gravitation

Applying the Law of Universal Gravitation to Determine Weight

Recall that on the surface of Earth, your weight is the product of your mass and the acceleration due to gravity at Earth's surface. It is also an application of Newton's second law. Your weight is always directed towards the mass that is causing the acceleration—in this case, Earth.

$$F = ma$$
$$W = mg$$

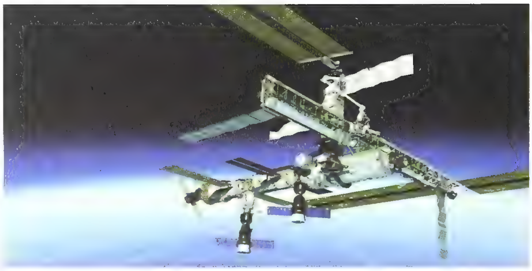
Away from Earth's surface, where the acceleration due to gravity is no longer equal to 9.81 m/s^2, your weight is best defined by an application of Newton's law of universal gravitation.

$$W = \frac{Gm_1m_2}{r^2}$$

Lesson 2 Lab: Newton's Law of Universal Gravitation

The International Space Station is an example of an object orbiting Earth. Earth's gravitational force helps keep the ISS in orbit.

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Weight and Orbits" in the search bar. Choose the item called "Weight and Orbits." The simulation used in this lab simulates the trajectory and gravitational force acting on a particle in Earth's gravitational field by varying its initial position and velocity.



© Stephen Strathdee/shutterstock

The International Space Station orbits Earth.

Problem

Why does an object's weight change if it moves away from Earth's surface?

Open the simulation; then continue with the Procedure section. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

Procedure

Once you have completed the lab, there is a Self-Check question. There are also lab questions to answer that you will submit to your teacher as part of your Module 4: Lesson 2 Assignment.

"Reset" (↻) the simulation, and set the distance scale to 100 pix = 10 000 km. Move the satellite to the North Pole by setting the *x* and *y* positions to 0 km and pressing "Enter." Display the "Data" box (▦).

Observation

Self-Check

SC 1. Record the weight, *W*, of the satellite shown at the bottom of the data box. What is the weight of the satellite at Earth's surface?

Check your work with the answer in the appendix.

Module 4: Lesson 2 Assignment

Remember to submit the answers to LAB 1 and LAB 2 to your teacher as part of your Lesson 2 Assignment in your Module 4 Assignment Booklet.

Analysis

LAB 1. Using $W = mg$, determine if the satellite's weight is consistent with the satellite's mass given as 40.0 kg and the acceleration due to gravity at the surface of the Earth (9.81 m/s^2).

Note: The acceleration due to gravity on Earth's surface is denoted *g* and is approximately constant at 9.81 m/s^2. In the simulation data box, the acceleration is denoted *a* and is not constant, as you will see, when you move the satellite away from the surface of Earth. Thus, the symbol *g* denotes a constant that applies at Earth's surface, and the symbol *a* denotes a variable (less than 9.81 m/s^2) for the acceleration due to gravity at any arbitrary point away from Earth.

LAB 2. What is the satellite's weight at a point one Earth radius above Earth's surface? On the simulation, position the satellite at $x = 0$ km and $y = 6378$ km. Enter these values in the position data entry fields, and then press "Enter." To view the satellite's new position, zoom out using the scale selections. Again, the weight is displayed at the bottom of the data box.

a. What is the weight of the satellite at one Earth radius from the surface?

b. If the satellite's weight changed but its mass remained constant, what variable has changed according to $W = ma$?

Read

Could the weight have been figured out without the simulation using Newton's law of universal gravitation? The formula looks more complex than many you have encountered, but with a calculator it is not too bad. Read from the middle of page 206 to the end of "Example 4.1" on page 207 of the physics textbook to see an example of how the law is used in calculations. Then you get a couple of chances to try it yourself.

Self-Check

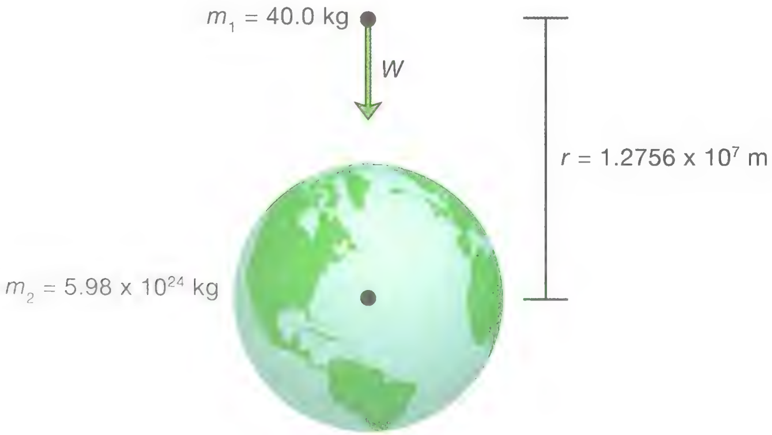
SC 2. What will be the weight of an 80.0-kg astronaut at a height of 1000 km above the surface of Earth? Use the data for the mass and radius of Earth from "Example 4.1" of the textbook.

Check your work with the answer in the appendix.

Module 4: Lesson 2 Assignment

Remember to submit the answer to LAB 3 to your teacher as part of your Lesson 2 Assignment in your Module 4 Assignment Booklet.

LAB 3. Away from the surface of Earth, $W = mg$ is no longer valid. You would need to use $W = ma$, assuming that you knew what the acceleration was at the satellite's position. (Remember that g is a constant 9.81 $\mathrm{m/s^2}$ and is only valid at Earth's surface.) Newton's law of universal gravitation is most useful away from the surface of Earth. Use this equation to determine if the satellite's weight in LAB 2 is consistent with the satellite's mass (40.0 kg) and the distance between the centre of Earth and the satellite. See the accompanying illustration.



Conclusion

The weight of the satellite at this altitude is 98.0 N, which is one-fourth of the weight at Earth's surface (392.2 N). An explanation for this reduction by a factor of 4 follows.

Apply Newton's law of universal gravitation. Find the magnitude of the force exerted by Earth on the satellite (i.e., the satellite's weight). This weight is inversely proportional to the square of the distance between Earth and the satellite. The distance has to be taken from the centre of Earth, not from the surface.

$$W_{\text{E}} = \frac{Gm_1 m_2}{(6\ 378\ \text{km})^2}$$

When the satellite is moved from Earth's surface (6378 km) to a point one Earth radius above Earth's surface (2 × 6378 km), the distance from Earth's centre is doubled.

$$W_{\text{o}} = \frac{Gm_1 m_2}{(2 \times 6\ 378\ \text{km})^2}$$

In Newton's universal law of gravitation, the distance *(r)* is squared and this means the force is only ¼ as large.

$$W_{\text{o}} = \frac{Gm_1 m_2}{4(6\ 378\ \text{km})^2}$$

$$W_{\text{o}} = \tfrac{1}{4}(W_{\text{E}})$$

Why do you have to use the distance from the centre of Earth, not from the surface?

Newton's law of universal gravitation makes a statement about the forces that two point-sized masses exert on each other. Since Earth certainly is not point-sized, you must calculate the force that this extended object exerts on a satellite. That is, you must sum all gravitational forces that the individual infinitesimal parts of the Earth exert on the satellite.

Another of Newton's accomplishments, calculus, can be used to simplify the calculation. You would likely make the assumptions that Earth is a perfect sphere and that its mass is distributed uniformly inside of it. Having made these assumptions, the result from calculus is remarkably simple. Earth acts like a point-sized particle with all of its mass at its centre.

Newton's law of universal gravitation is also useful for determining the force of attraction between any two masses. Consider the force that keeps the Moon in orbit. For example, using the data table below, calculate the force of attraction between the Moon and Earth.

mass of Earth = 5.98×10^{24} kg	mass of the Moon = 7.34×10^{22} kg
mean lunar orbit = 3.8×10^{8} m	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$

$$F_g = \frac{Gm_1 m_2}{r^2}$$

$$F_g = \frac{\left(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(7.34\times10^{22}\ \text{kg}\right)}{\left(3.8\times10^{8}\ \text{m}\right)^2}$$

$$F_g \doteq 2.027\,476\,759\times10^{20}\ \text{N}$$

$$F_g = 2.0\times10^{20}\ \text{N, correct to 2 significant digits}$$

This very large force keeps the Moon in its orbit!

Module 4: Lesson 2 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 2 Assignment in your Module 4 Assignment Booklet.

Try This

TR 1. Determine the gravitational force of attraction between a small cargo ship (5.00×10^4 kg) and a supertanker (1.50×10^6 kg) that is 500 m away. Compare your answer to the force of attraction between Earth and the Moon. Why are gravitational forces between masses on Earth so small?

Read

Read "Newton's Law of Universal Gravitation" on pages 203 and 204 of the physics textbook to see a way of calculating the gravitational force.

Self-Check

SC 3. Complete the "Concept Check" question on page 205 of the textbook.

Check your work with the answer in the appendix.

Read

Read "Using Proportionalities to Solve Gravitation Problems" on pages 207 and 208 of the physics textbook to see how the concepts from the previous reading and question are applied.

Self-Check

SC 4. Complete “Practice Problem” on page 208 of the textbook.

Check your work with the answer in the appendix.

Read

What happens when two widely separated objects both exert a gravitational pull on a third object? Read “Using Superposition to Find the Net Gravitational Force on an Object” on pages 208 to 210 of the physics textbook.

Self-Check

SC 5. Complete question 5 of “4.2 Check and Reflect” on page 215 of the textbook. This is not a superposition question, although it has similarities.

Check your work with the answer in the appendix.

Applying the Law of Universal Gravitation to Determine Acceleration

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms “weight and orbits” in the search bar. Choose the item called “Weight and Orbits” to complete the following questions.

Self-Check

SC 6. What happens to the magnitude of the acceleration as the satellite is moved farther away from Earth? Investigate this by turning on the acceleration vector by clicking the “Vectors” button (/) and selecting acceleration and deselecting velocity. Then click off the “Vectors” button. Set the scale setting to 100 pix = 10 000 km. Observe how the acceleration vector changes by dragging the satellite to various positions around Earth.

What do you notice about the magnitude and direction of the acceleration vector?

SC 7. On the simulation, set the scale setting to 100 pix = 10 km. This is the most magnified setting. The satellite will be outside of the bounds of the window. Set $x = 0$ km and $y = 0$ km to move the satellite back to the North Pole. Now drag the satellite around the display area, and observe the changes in the magnitude of the acceleration vector. Are the changes in magnitude large or small near Earth's surface?

Check your work with the answer in the appendix.

You can derive a general expression for the magnitude of the acceleration (a) acting on a satellite of mass (m_1). Use Newton's second law ($F = ma$) and the law of universal gravitation ($F_g = \frac{Gm_1m_2}{r^2}$). You'll also need Earth's mass, m^2, and the distance (r) of the satellite from the centre of Earth.

$$
\begin{aligned}
F &= \frac{Gm_1m_2}{r^2} \\
m_1a &= \frac{Gm_1m_2}{r^2} \quad \text{(remembering that } F = ma\text{)} \\
a &= \frac{Gm_2}{r^2}
\end{aligned}
$$

Using this equation, it is possible to determine the local value for the acceleration due to gravity at any point relative to a large mass, such as a comet, moon, planet, or star.

Read

Read "Relating Gravitational Field Strength to Gravitational Force" on pages 216 to 219 of the physics textbook to prepare for the following questions.

Module 4: Lesson 2 Assignment

Remember to submit the answers to TR 2 and TR 3 to your teacher as part of your Lesson 2 Assignment in your Module 4 Assignment Booklet.

Try This

TR 2. Determine the acceleration on the surface of the Moon if it has a radius of 1.74×10^6 m and a mass of 7.35×10^{22} kg. Explain how this would affect the walking pattern of a human being.

TR 3. Determine the acceleration of a satellite that is orbiting Earth at an altitude of 600 km. Use a mass of 5.98×10^{24} kg and a radius of 6.378×10^6 m for Earth. Use the simulation to verify your answer. Set the y position to 600 km, and look for the acceleration value in the data box.

Read

Why isn't the value of g exactly 9.81 m/s^2 everywhere on the surface of Earth? Read "Different Values of Gravitational Field Strength on Earth" on page 221 of the textbook. Then read "Applications of the Variation in g in Geology" beginning on page 222. Look for the answers to this question in the next Self-Check question.

Self-Check

SC 8. What are four factors that can slightly change the value of g at the surface of Earth?

Check your work with the answer in the appendix.

Module 4: Lesson 2 Assignment

Remember to submit the answer to TR 4 to your teacher as part of your Lesson 2 Assignment in your Module 4 Assignment Booklet.

Try This

TR 4. Complete question 1 of the "Practice Problems" on page 220 of the textbook.

Module 4: Lesson 2 Assignment

Remember to submit Reflect and Connect to your teacher as part of your Lesson 2 Assignment in your Module 4 Assignment Booklet.

Reflect and Connect

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Is the Moon falling, just like an apple from a tree? Newton's law of universal gravitation would have you believe just that, yet the motion of the Moon appears to contradict the definition of acceleration for a falling apple. Both the apple and the Moon are under the influence of the same gravitational field, yet they appear to move very differently. View the following two video clips to compare and contrast the motion of a falling apple with the motion of the Moon. (**Note:** You may be required to login with a username and a password. Contact your teacher for this information.)

Go to the Physics 20 Multimedia DVD, and view the video clip titled "Moon Orbit 1."

View the video clip titled "Moon Orbit 2." Half way through the clip, it's assumed that you know the formula for circular acceleration, $a = \frac{v^2}{r}$. Don't be confused by this; you will work with the formula in Module 5.

Choose from one of the following activities to identify and explain the contradiction you have witnessed.

- Create a graphic representation of the contradiction with labels.
- Create a model of what is happening.
- Explain the contradiction through mathematics.
- Create a written scenario identifying what would need to change for the Moon to fall into Earth and the consequences if this were to happen.

When you are finished, place your work in your Physics 20 course folder and submit your response to your teacher for marks.

Discuss

© Photodisc/Getty Images

One day, so the story goes, Isaac Newton was under an apple tree reading a book when an apple fell and struck his head. Newton wondered what made the apple fall. Until Newton, many people believed that Earth and the heavens were governed by two totally dissimilar sets of laws or principles. But Newton, after observing a falling apple and the Moon, wondered if both the Moon and the apple were falling towards Earth. In other words, maybe a *universal* set of laws existed that would describe motion on Earth *and* in the heavens.

Humankind's journey to the Moon began with the work of Nicolaus Copernicus and was made possible by the discoveries of Newton. At the age of 22, Newton retreated to his family farm to escape the black plague. By consulting known data about the Moon's orbit and other variables, he developed a relationship that may be used to describe the force of attraction between any two bodies in the universe. Newton published these laws in 1687, in the book titled *Mathematical Principles of Natural Philosophy*. His work is one of the greatest scientific books ever published.

Here is an excerpt from his book:

"The same year I began to think of gravity extending to ye orb of the Moon, and . . . from Kepler's Rule (Kepler's third law) . . . I deduced that the forces which keep the Planets in their Orbs must (vary) reciprocally as the square of their distances from the centres about which they revolve: and thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded Mathematicks and Philosophy more than at any time since."
—Isaac Newton

Newton could not have developed his theory in isolation. Great scientific discoveries are made with the contributions of many individuals. Johannes Kepler is mentioned specifically. See an historical account of Newton's discovery in Newton's Perspective. It takes a long time to load, so be patient. (**Note:** You may be required to login with a username and a password. Contact your teacher for this information.)

Using the discussion forum, describe one of the following:
- the contributions of Kepler and Galileo
- how Newton used Kepler's and Galileo's observations in a fundamentally new way
- why Newton's law of universal gravitation was a significant step forward relative to the work of Kepler and Galileo

When you are finished, place your work in your Physics 20 course folder.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider travel to, exploration of, or study of space. To help you consolidate your learning from this lesson, complete at least one of these reflection activities. You may choose to do one of these activities with a partner. If you work with a partner, be sure to add both your names to the reflections.
- Create an advertising campaign to sell the universal law of gravitation to a group of younger students or the general public. Choose your media—print, posters, audio, video, or computer multimedia—and sell, sell, sell!

- Imagine that you are a guest speaker in a Grade 9 science class and you have been asked to explain the way the law of universal gravitation affects the lives of the students. Prepare a short presentation that you might use. For example, satellite radio, TV, GPS, and the seasons might be included in your message. Your presentation can be in any form. You may choose to write a report, create diagrams, record the presentation, or use PowerPoint.

Store your completed reflection in your Physics 20 course folder.

Module 4: Lesson 2 Assignment

Make sure you have completed all of the questions for the Lesson 2 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 4 assignments have been completed.

Lesson Summary

In this lesson you explored the following questions:
- How does gravity affect planets, moons, and other objects in the solar system?
- What does the law of universal gravitation tell you about gravity?
- Is the force of gravity a constant?

Newton's law of universal gravitation can be used to determine the following:
- an object's weight at any point relative to the source of the gravitational field
- the acceleration due to gravity at any point relative to the source of a gravitational field
- the force of attraction between any two masses

The universal nature of this law is useful for understanding planetary and satellite motion away from the surface of Earth.

Newton's law of universal gravitation states the gravitational force of attraction between any two masses is proportional to the product of the two masses and inversely proportional to the square of the distance between them. It is summarized mathematically as

$$F_g = \frac{Gm_1 m_2}{r^2}$$

Quantity	Symbol	SI Unit
force of gravity	F_g	N
mass	m	kg
separation between centres of masses	r	m
gravitational constant	G	$G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg}\bullet\text{s}^2}$ or $G = 6.67 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$

Applying Newton's second law to the equation gives the following expression for the acceleration due to gravity (a) at a distance (r) from the centre of the mass (m_2) creating the field.

$$m_1 a = \frac{Gm_1 m_2}{r^2}$$

$$a = \frac{Gm_2}{r^2}$$

Lesson Glossary

Newton's law of universal gravitation: a law that states the gravitational force of attraction between any two masses is proportional to the product of the two masses and inversely proportional to the square of the distance between them

It is summarized mathematically as

$$F_g = \frac{Gm_1 m_2}{r^2}$$

universal gravitational constant: the value G in the equation $F_g = \frac{Gm_1 m_2}{r^2}$ summarizing Newton's law of universal gravitation $G = 6.67 \times 10^{-11}\ \frac{\text{N} \bullet \text{m}^2}{\text{kg}^2}$.

Lesson 3—Universal Gravitational Field Theory and the Density of Earth

Get Focused

Earth has a mass of 5.9725 billion trillion tonnes. If scientists didn't know this, they wouldn't be able to explore the surface and geology of other planets, such as Mars. This should bring to mind two questions. First, how do scientists know how much mass Earth has (since it can't be put onto a giant scale)? Second, how is that related to the development and deployment of rovers used to explore other planets, such as the rovers *Spirit* and *Opportunity* currently on Mars?

© Michael Ransburg/shutterstock

The Mars Rover was designed by NASA to explore the surface of Mars.

In the previous lesson you learned that a gravitational force is often referred to as a mass-attracting force. It is a force that exists between any two masses. The force can tell something about the masses causing it. But exactly how are the strength of a gravitational field and the mass creating it related? How could this relationship be investigated in a laboratory setting? Finally, how has this understanding led to the exploration of other planets and moons?

In this lesson you will explore the following questions:

- Is the force of gravity a constant?
- How do scientists calculate weight on other planetary objects, such as the Moon?
- How does the law of universal gravitation explain the motion of orbiting bodies?
- How can you calculate the mass of Earth?

Module 4: Lesson 3 Assignments

In this lesson you will complete the Lesson 3 Assignment in the Module 4 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, and TR 4
- Reflect and Connect

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

Henry Cavendish used a torsion balance in his experiments.

Earth was weighed in 1798 by Henry Cavendish. In doing so, he experimentally verified Newton's law of universal gravitation. Until this had been done, the exact mathematical relationship between the force of gravity and the mass causing it was unknown.

What was known was that the force of gravity varied with the product of the masses and varied inversely to the square of the distances separating them. Describing this mathematically produces the following proportionality:

$$F_g \propto \frac{m_1 m_2}{r^2}$$

proportionality constant: the constant value of the ratio of two variables

In order to make this an equation, a **proportionality constant** is introduced. In this case, the constant is the universal gravitational constant (G). As you saw in the last lesson, it is related to the force of attraction between two masses and the distance separating them, as follows:

$$F_g = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$G = \frac{F_g r^2}{m_1 m_2}$$

The Cavendish Experiment

torsion balance: a measuring instrument designed to measure small forces by the torsion (twisting) they cause in a thin wire

Based on this relationship, Cavendish set out to measure the force of attraction between two large metal balls of known mass separated by a known distance. It was clear that the force of attraction would be very small. (Remember, the force of attraction between two masses is not significant until the masses are extremely large.) A **torsion balance** was used to accurately measure the small forces that were expected. A torsion balance is a device that measures the amount of "twisting" that occurs along a suspension fibre or cable. It had been invented 20 years before Cavendish performed his experiment.

By measuring the force of attraction between two large sets of lead balls with a given mass and distance of separation, Cavendish determined that the value of the universal gravitational constant is

$$G = 6.673 \times 10^{-11}\ \frac{\text{m}^3}{\text{kg}\bullet\text{s}^2}\ .$$

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the "Cavendish" video to see one method of weighing Earth.

Read

Read about Cavendish's experiment in "Determining the Value of the Universal Gravitational Constant" on pages 205 and 206 of the physics textbook.

Module 4: Lesson 3 Assignment

Remember to submit the answers to TR 1 and TR 2 to your teacher as part of your Lesson 3 Assignment in your Module 4 Assignment Booklet.

Try This

TR 1. Explain how Cavendish was able to determine the force of attraction in his experimental apparatus.

TR 2. The mass of each lead ball in Cavendish's experiment was 195 kg. Given the value of G, what would be the expected force of attraction between two of these balls if their centres were separated by 1.0400 m?

Measuring the Mass of Earth

By determining the value for the universal gravitational constant, Cavendish indirectly measured the mass of Earth based on the following mathematical relationships:

$$F_g = m_1 g$$

$$F_g = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$m_1 g = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$g = \frac{Gm_2}{r^2}$$

$$m_2 = \frac{gr^2}{G}$$

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The method Cavendish used is still the method used for finding Earth's weight accurately. You can read about the results of a recent experiment on the web. Use the following keywords to search for information on the Internet: "space," "physicists," "forces," "Earth," "drop," and "weight."

Module 4: Lesson 3 Assignment

Remember to submit the answer to TR 3 to your teacher as part of your Lesson 3 Assignment in your Module 4 Assignment Booklet.

Try This

TR 3. Given the value for the universal gravitational constant, the average value for the acceleration due to gravity at the surface of Earth, and the radius of Earth, determine Earth's mass. (Earth's radius can be found on the physics data sheet in the Constants section along with other useful information.)

Constants

Acceleration Due to Gravity Near Earth	$\left\lvert \vec{a}_g \right\rvert = 9.81\,\text{m/s}^2$
Gravitaional Constant	$G = 6.67 \times 10^{-11}$ N•m^2/kg^2
Radius of Earth	$r_e = 6.37 \times 10^6$ m
Mass of Earth	$M_e = 5.98 \times 10^{24}$ kg
Elementary Charge	$e = 1.60 \times 10^{-19}$ C
Coulomb's Law Constant	$k = 8.99 \times 10^9$ N•m^2/C^2
Electron Volt	$1\text{eV} = 1.60 \times 10^{-19}$ J
Index of Refraction of Air	$n = 1.00$
Speed of Light in Vacuum	$c = 3.00 \times 10^8$ m/s
Planck's Constant	$h = 6.63 \times 10^{-34}$ J.s $h = 4.14 \times 10^{-15}$ eV.s
Atomic Mass Unit	$u = 1.66 \times 10^{-27}$ kg

Self-Check

SC 1. Given the mass of the Moon is 7.3477×10^{22} kg and it has a mean radius of 1737.103 km, determine the value for the acceleration due to gravity on the surface. Explain how this would affect your walking speed.

Check your work with the answer in the appendix.

Module 4: Lesson 3 Assignment

Remember to submit the answer to TR 4 to your teacher as part of your Lesson 3 Assignment in your Module 4 Assignment Booklet.

Try This

TR 4. Density is a measure of the mass per unit of volume for a material. Mathematically, this can be expressed in units of kg/m^3. Given that the equation for the volume of sphere is based on radius, determine the density of Earth and compare it to that of water (998 kg/m^3 at 20ºC). Explain how this helps people understand Earth's interior.

density: the mass per unit volume of an object

$$\text{volume} = \frac{4}{3}\pi r^3$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Self-Check

SC 2. Given the mass of Neptune is 1.03×10^{26} kg and that it has an equitorial radius of 2.48×10^7 m, determine the value for the acceleration due to gravity on the equator of Neptune.

Check your work with the answer in the appendix.

Density is a measure of the mass per unit of volume for a material. Mathematically, this can be expressed in units of kg/m3. Given that the equation for the volume of sphere is based on radius, determine the density of Earth and compare it to that of water (998 kg/m3 at 20ºC). Explain how this helps people understand Earth's interior.

$$\text{volume} = \frac{4}{3}\pi r^3$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Self-Check

SC 3. Complete question 2 of "Practice Problems" on page 207 of the physics textbook.

Check your work with the answer in the appendix.

Read

Read "The Role of Gravitational Force on Earth's Tides" on pages 210 to 213 of the physics textbook.

Self-Check

SC 4. The Moon can cause a high tide on the side of Earth nearest the Moon. How does the Moon cause a high tide on the side of Earth farthest away from the Moon? (See side B in "Figure 4.24" on page 211 of your textbook.)

Check your work with the answer in the appendix.

Read

Read "The Role of Gravitational Force on Interplanetary Travel" on pages 213 and 214 of the physics textbook.

Self-Check

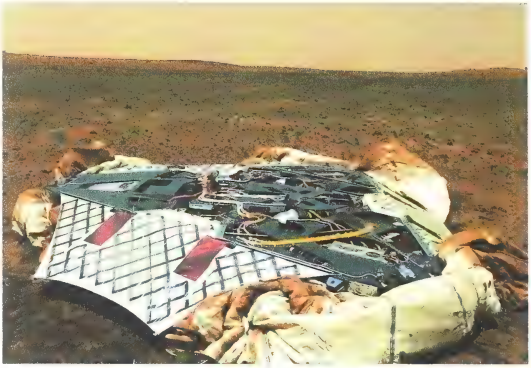
SC 5. How is gravity used to accelerate a spacecraft in its flight?

Check your work with the answer in the appendix.

Module 4: Lesson 3 Assignment

Remember to submit the answer to Reflect and Connect to your teacher as part of your Lesson 3 Assignment in your Module 4 Assignment Booklet.

Reflect and Connect



Courtesy NASA

The Mars Rover transmitted this image of the landing platform after it landed on Mars.

On January 4, 2004, the *Spirit* rover landed on Mars. Three weeks later, on January 25, 2004, the *Opportunity* rover landed on the other side of the planet. See the view from the *Opportunity* spacecraft as it approached Mars in the short Lesson 3 Reflect and Connect video.

Both *Opportunity* and *Spirit* successfully landed on the planet and were able to drive off their platforms and explore the surface for many years.

In the planning stages of this mission, scientists had to engineer a delivery system that would be able to descend in the gravitational field surrounding Mars and ensure the rovers were not damaged when they landed. NASA would also need to design a locomotion system that would allow the rovers to move over the surface of the planet. By April 4, 2006, *Spirit* had lasted almost 9 times longer than planned, and the motors on *Spirit*'s wheels had rotated more than 13 million times, exceeding the design standards by an incredible amount. At this point, one of the six wheels on the rover had failed and questions began about its ability to continue to drive successfully on five wheels while dragging its sixth wheel in the sand. As of March 2007, both rovers continue to operate on Mars.

In a couple of paragraphs, explain how NASA engineers applied Cavendish's work and Newton's law of universal gravitation when the following tasks were completed:

- The spacecraft had to approach Mars at a very high rate of speed while being pulled in by an increasingly strong gravitational field. Scientists needed to slow the spacecraft down considerably before it reached the atmosphere and slow the spacecraft down some more again before it reached the surface.
- The rover had to survive a direct impact when it reached the surface.
- The rover's locomotion system had to support its weight on Mars, on a relatively unknown terrain with hills and rocks.
- The rover's locomotion system had to operate on minimal power.

You can learn all about the rovers at the NASA website. You can find the website using a search engine.

Discuss

The work of Newton and Cavendish helped scientists understand how to get to the Moon and what to expect when landing. Humans landed on the moon six times, starting with *Apollo 11* in 1969 and ending with *Apollo 17*, which left the Moon on December 19, 1972. You can learn about the *Apollo* missions at the NASA website.

Courtesy NASA

Humankind has not been to the Moon since 1972, but now there are plans for a permanent human presence on Earth's closest neighbour.

This has led to questions:

- Why would scientists want to go back to the Moon?
- Why would scientists want to go to Mars?

NASA, in collaboration with 13 other international space agencies, has started to answer those questions. Six general themes emerged in the answers.

In the discussion forum, prepare your best answer to the previous questions. Choose any two of the following six themes, which were identified by NASA as reasons for exploring the Moon and Mars. Describe what it would mean if humans and robots established a permanent settlement on the Moon:

- human civilization
- scientific knowledge
- global partnership
- economic expansion
- public engagement
- exploration preparation

Place your finished work in your Physics 20 course folder.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will ask you to consider travel to, exploration of, or study of space. To help you consider the concepts from this lesson, complete at least one of these activities:

- Explore the role of remote-controlled space exploration. How do costs, challenges, advantages, and disadvantages affect which technologies are used to overcome the challenges of gravity and the inhospitality of space?

- Looking back on the impact on the world of *Sputnik* and the lunar race, where do you think space travel and exploration is headed in the next decade? What is different between the present and when *Sputnik* started the space race? What will drive the economic investment needed for future space travel and exploration? How is it possible you could benefit?

Store your completed reflection in your Physics 20 course folder.

Module 4: Lesson 3 Assignment

Make sure you have completed all of the questions for the Lesson 3 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 4 assignments have been completed.

Lesson Summary

In this lesson you explored the following questions:

- Is the force of gravity a constant?
- How do scientists calculate weight on other planetary objects, such as the Moon?
- How does the law of universal gravitation explain the motion of orbiting bodies?
- How can you calculate the mass of Earth?

By measuring the force of attraction between two masses using a torsion balance, Cavendish determined the value for the gravitational constant (*G*) and, indirectly, also weighed Earth.

$$F_g = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$G = \frac{F_g r^2}{m_1 m_2}$$

$$G = 6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$$

$$m_{\text{earth}} = \frac{g\left(r_{\text{earth}}\right)^2}{\text{G}}$$

$$m_{\text{earth}} = \frac{\left(9.81\ \text{m/s}^2\right)\left(6.37 \times 10^6\ \text{m}\right)^2}{6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}}$$

$$m_{\text{earth}} \doteq 5.967\ 906\ 882 \times 10^{24}\ \text{kg}$$

$$m_{\text{earth}} = 5.98 \times 10^{24}\ \text{kg, correct to 3 significant digits}$$

$$g = \frac{Gm_{\text{E}}}{r_{\text{E}}^2}$$

$$m_{\text{E}} = \frac{g\ r_{\text{E}}^2}{G}$$

$$m_{\text{E}} = \frac{\left(9.81\ \text{m/s}^2\right)\left(6.37 \times 10^6\ \text{m}\right)^2}{6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}$$

$$m_{\text{E}} = 5.97 \times 10^{24}\ \text{kg}$$

Cavendish's work verified Newton's law of universal gravitation and, in doing so, enabled us to understand gravitational fields beyond Earth's surface. This allows us to calculate the weight of objects on the surface of other planets and to understand how the force of gravity changes as you approach or move away from a very large mass, such as a planet.

Lesson Glossary

density: the mass per unit volume of an object

proportionality constant: the constant value of the ratio of two variables

torsion balance: a measuring instrument designed to measure small forces by the torsion (twisting) they cause in a thin wire

Module 4—Gravitational Force

Module Summary

In this module you have studied some ways of applying Isaac Newton's laws, and you have learned about his law of universal gravitation. You saw how this law can help you explain the motions of astronomical objects.

People often confuse the terms *weight* and *mass*. Remember that mass is a measure of the amount of matter that makes up an object, whereas weight is a product of mass and the acceleration due to gravity.

Weight is expressed in the following equation:

$$\vec{W} = m\vec{g}$$

Quantity	Symbol	SI Unit
weight	$\vec{W}$	N
mass	m	kg
acceleration due to gravity	$\vec{g}$	m/s^2

You can use Newton's law of universal gravitation to help you understand how gravitational fields function. The field strength (g) increases as you approach the mass. For Earth, the maximum force is 9.81 m/s^2 directed toward the centre of Earth. The field strength (g) obeys an inverse square law $\left(g \propto \frac{1}{d^2}\right)$ and extends outwards, affecting the motion of all objects that encounter it.

Newton's law of universal gravitation states that the gravitational force of attraction between any two masses is proportional to the product of the two masses and inversely proportional to the square of the distance between them.

Expressed as an equation, it is as follows:

$$F_g = \frac{Gm_1m_2}{r^2}$$

Quantity	Symbol	SI Unit
force of gravity	F_g	N
mass	m	kg
separation between centres of masses	r	m
gravitational constant	G	$G = 6.67 \times 10^{-11}\ \frac{\text{m}^3}{\text{kg}\bullet\text{s}^2}$ or $G = 6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$

You can use Newton's law of universal gravitation to find the following:

- an object's weight at any point relative to the source of the gravitational field
- the acceleration due to gravity at any point relative to the source of a gravitational field
- the force of attraction between any two masses

By measuring the force of attraction between two masses using a torsion balance, Henry Cavendish determined the value for the gravitational constant (G).

$$F_g = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$G = \frac{F_g r^2}{m_1 m_2}$$

$$G = 6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$$

This let Cavendish calculate the mass of Earth. Start with the equation $g = \frac{Gm}{r^2}$, and solve it for mass $\left(m = \frac{gr^2}{G}\right)$.

$$m_\text{E} = \frac{g\left(r_\text{E}\right)^2}{\text{G}}$$

$$m_\text{E} = \frac{\left(9.81\ \text{m/s}^2\right)\left(6.37 \times 10^6\ \text{m}\right)^2}{6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}}$$

$$m_\text{E} \doteq 5.967\ 906\ 882 \times 10^{24}\ \text{kg}$$

$$m_\text{E} = 5.97 \times 10^{24}\ \text{kg, correct to 3 significant digits}$$

Cavendish experimentally verified Newton's law of universal gravitation and, in doing so, enabled us to understand gravitational fields beyond the surface of Earth. This will help you as you continue on to the next module.

Module Assessment

The assessment for Module 4 consists of three (3) assignments, as well as a final module inquiry project.

- Module 4: Lesson 1 Assignment
- Module 4: Lesson 2 Assignment
- Module 4: Lesson 3 Assignment
- Module 4 Project

Module 4 Project

The Module 4 Project is to choose **one** of the following items from your Physics 20 course folder:

- Going Beyond item from Lesson 1
- Reflect on the Big Picture item from Lesson 2
- Reflect on the Big Picture item from Lesson 3

You are then to submit your one item to your teacher for grading. Along with the item from the course folder, include an explanation about why you chose that particular item to submit.

Module 4 Glossary

density: the mass per unit volume of an object

gravitational field: a field created by any object with mass, extending outward in all directions, which determines the influence of that object on all others

gravity: a force of attraction towards the centre of objects

inverse square law: any physical law stating that some quantity is inversely proportional to the square of the distance from a point

mass: the property of physical objects that, roughly speaking, measures the amount of matter they contain

Newton's law of universal gravitation: summarized mathematically as $F_g = \frac{Gm_1 m_2}{r^2}$

proportionality constant: the constant value of the ratio of two proportional quantities

torsion balance: a measuring instrument designed to measure small forces by the torsion (twisting) they exert on a thin wire

universal gravitational constant: the value G in the equation $F_g = \frac{Gm_1 m_2}{r^2}$ summarizing Newton's law of universal gravitation

Unit B Conclusion

In Module 3 you saw that a non-zero (unbalanced) net force will cause a change in velocity. You examined Newton's first law of motion to help you understand the forces acting on an object at rest and an object with uniform motion. Newton's second law of motion helped explain the relationships among net force, mass, and acceleration. You applied Newton's third law of motion to interactions between two objects and recognized that the two equal and opposite forces act on different bodies. You examined the differences between static and kinetic forces of friction. You were able to apply vector tools from Unit A to calculate the resultant force acting on an object. Finally, you applied Newton's laws of motion to solve linear-motion problems in horizontal, vertical, and inclined planes near the surface of Earth.

From this, you can see how understanding forces can help humans improve and change our environment. Many improvements in transportation have been brought about through technological advancements that reflect this understanding. Seatbelts, timing of traffic lights, runway design for airplanes, and space travel are only a few areas where the applications of the principles of dynamics have led to improvements and the development of new mechanical technologies.

In Module 4 you saw that the gravitational force is one of the fundamental forces in nature. You examined the principles of the Cavendish experiment and studied Newton's law of universal gravitation and used it to relate the gravitational constant to the local value of the acceleration due to gravity. You defined field as a concept that replaces action at a distance. Finally, you predicted differences in weights of objects on different planets.

You should now have a clearer picture of gravitational effects in the universe. You can distinguish between mass and weight in space or on Earth. You can see the relationship between acceleration due to gravity and gravitational field strength. The concepts, models, and theories studied in this module are often used to interpret and explain observations and to predict future observations.

The understanding of dynamics and kinematics that you gained in Units A and B will form a strong foundation for the new concepts related to motion you will explore in Unit C.

Unit B Assessment

A roofer is dragging a 15.0-kg bundle of shingles up the roof using a light rope parallel to the roof. The roof is sloped at an angle of 30.0° with the horizontal.

a. Draw a free-body diagram of the forces involved.

b. What is the force component perpendicular to the roof of the weight of the bundle of shingles?

c. What is the magnitude of the force of friction on the bundle of shingles as it is pulled up the roof? The coefficient of kinetic friction of the shingle bundle on the roof is 0.520.

d. Calculate the force parallel to the roof with which the roofer must pull on the rope to move the shingles at constant speed up the roof.

e. Suppose an astronaut was building a similar structure on Mars. What would be the force component perpendicular to the roof of the weight of a 15.0-kg bundle of materials on the surface of Mars? The mass of Mars is 6.42×10^{23} kg, and the radius of Mars is 3.40×10^{6} m.

f. Between Mars and Jupiter, the asteroid Ceres orbits the Sun at an average radius of 2.766 AU. Use Kepler's third law to calculate the time in Earth years it takes for Ceres to make one complete orbit.

Submit your answers to the Unit B Assessment to your teacher for marks.

Self-Check Answers

Lesson 1

SC 1.

Diagram (b) best represents the gravitational forces because it not only shows attraction, it also shows the equal and opposite forces predicted by Newton's third law.

SC 2.

(a) The gravitational field strength will be increased by a factor or 16 because it is inversely proportional to the radius.

(b) The gravitational field strength will be 1/4 of its original value.

(c) The gravitational field strength remains the same because both m and F_g are doubled, so the ratio remains the same.

(d) The gravitational field strength remains the same because both m and F_g are halved, so the ratio remains the same.

Lesson 2

SC 1. The weight of the satellite at Earth's surface is **393.2 N**.

SC 2.

Given

$m_a = 80.0$ kg
$m_E = 5.97 \times 10^{24}$ kg
$r_E = 6.38 \times 10^6$ m
$h = 1000$ km $= 1.000 \times 10^6$ m

Required

the weight of the astronaut ($\vec{F}_g$)

Analysis and Solution

The distance of the astronaut from the centre of Earth will be (6.38×10^6 m + 1.000×10^6 m) or 7.38×10^6 m. The weight of the astronaut will be the force of gravity on the astronaut, which can be calculated using Newton's law of universal gravitation.

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(80.0\ \text{kg})\left(5.97\times10^{24}\ \text{kg}\right)}{\left(7.38\times10^{6}\ \text{m}\right)^2}$$

$$= 585\ \text{N}$$

Paraphrase

The weight of the astronaut will be 585 N.

SC 3.

(a) The magnitude of the gravitational force increases. It becomes 4 times the original force.
(b) The magnitude of the gravitational force increases. It becomes 4 times the original force.
(c) The magnitude of the gravitational force decreases. It becomes 1/16 of the original force.

SC 4.

(a) If the separation distance increases to 4/3 of its original value, the magnitude of the gravitational force will change by a factor of $(3/4)^2$ or become 9/16 of its original value. If the m_A increases to 3/2 of its original value, the magnitude of the gravitational force will become 3/2 of its original value. If the m_B decreases to 1/2 of its original value, the magnitude of the gravitational force will become 1/2 of its original value. The net effect of all three changes is that the gravitational force will become (5.9×10^{-11} N)(9/16)(3/2)(1/2) or 2.5×10^{-11} N.

(b) If the separation distance decreases to 1/6 of its original value, the magnitude of the gravitational force will change by a factor of $(6)^2$ or become 36 times its original value. If the m_A decreases to 1/2 of its original value, the magnitude of the gravitational force will become 1/2 of its original value. If the m_B increases to 5/4 of its original value, the magnitude of the gravitational force will become 5/4 of its original value. The net effect of all three changes is that the gravitational force will become (5.9 × 10–11 N)(36)(1/2)(5/4) or 1.3×10^{-9} N.

SC 5.

(a) Phobos will exert a greater gravitational force on Mars because Phobos is closer to Mars and has a larger mass.

(b)

$$\begin{aligned}F_{\text{g Deimos}} &= \frac{Gm_1m_2}{r^2}\\ &= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.8\times10^{15}\ \text{kg}\right)\left(6.42\times10^{23}\ \text{kg}\right)}{\left(2.3\times10^{7}\ \text{m}\right)^2}\\ &= 1.5\times10^{14}\ \text{N}\end{aligned}$$

$$\begin{aligned}F_{\text{g Phobos}} &= \frac{Gm_1m_2}{r^2}\\ &= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.1\times10^{16}\ \text{kg}\right)\left(6.42\times10^{23}\ \text{kg}\right)}{\left(9.4\times10^{6}\ \text{m}\right)^2}\\ &= 5.3\times10^{15}\ \text{N}\end{aligned}$$

SC 6. The acceleration vector has greater magnitude close to Earth and rapidly decreases in magnitude as it is moved away. It always points toward the centre of Earth.

SC 7. You should find that there is very little change in magnitude and no noticeable change in the direction of the acceleration. How do you explain this? At this scale setting, the distance changes that are possible within the window are just tiny fractions of the radius of Earth. For a significant change in the acceleration to occur, the distance from Earth's surface must change by an amount comparable to the radius of Earth.

SC 8. Factors that can slightly change the value of g at the surface of Earth include latitude, altitude, the density of Earth's crust at that location, and the rotation of Earth.

Lesson 3

SC 1.

$$\begin{aligned}
F_g &= \frac{Gm_1m_2}{r^2} \\
F_g &= m_1g \\
m_1g &= \frac{Gm_1m_2}{r^2} \\
g &= \frac{Gm_2}{r^2} \\
g &= \frac{\left(6.673\times10^{-11}\ \text{m}^3/\text{kgs}^2\right)\left(7.35\times10^{22}\ \text{kg}\right)}{\left(1.73\times10^{6}\ \text{m}\right)^2} \\
g &= 1.64\ \text{m/s}^2
\end{aligned}$$

Your walking speed would be increased as your weight would be reduced to 1/6th its value on Earth, which would make motion easy.

SC 2.

$$\begin{aligned}
F_g &= \frac{Gm_1m_2}{r^2} \\
F_g &= m_1g \\
m_1g &= \frac{Gm_1m_2}{r^2} \\
g &= \frac{Gm_2}{r^2} \\
g &= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.03\times10^{26}\ \text{kg}\right)}{\left(2.48\times10^{7}\ \text{m}\right)^2} \\
g &= 11.2\ \text{m/s}^2
\end{aligned}$$

SC 3.

Given

$m_T = 4.6 \times 10^7$ kg
$F_g = 61$ N
$r = 100$ m

Required

the mass of the iceberg (m_i)

Analysis and Solution

Assume that the distance is between the centres of the *Titanic* and the iceberg. Use Newton's law of universal gravitation and rearrange the equation to solve for mass.

$$F_g = G\frac{m_I m_T}{r^2}$$

$$\begin{aligned} m_I &= \frac{F_g r^2}{Gm_T} \\ &= \frac{(61\,\text{N})(100\,\text{m})^2}{\left(6.67\times10^{-11}\,\frac{\text{N"m}^2}{\text{kg}^2}\right)\left(4.6\times10^7\,\text{kg}\right)} \\ &= 2.0\times10^8\,\text{kg} \end{aligned}$$

Paraphrase

The mass of the iceberg was 2.0×10^8 kg.

SC 4. The high tide on the side farthest from the Moon is caused by the Moon attracting Earth by gravity more than the water, which is farther away, and pulling Earth closer to itself, leaving the ocean water behind.

SC 5. If a spacecraft approaches a planet travelling in the same direction, the planet's gravitational attraction will accelerate the planet toward it. This curves the spacecraft's path in, and the acceleration slingshots the craft away at a different trajectory with increased speed.

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